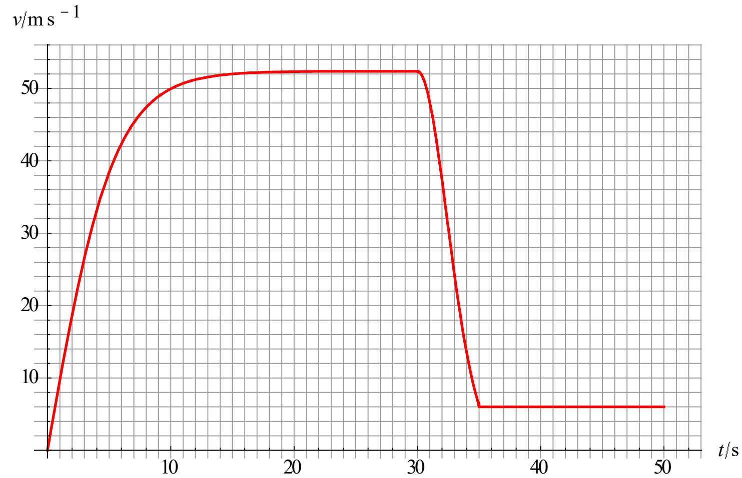
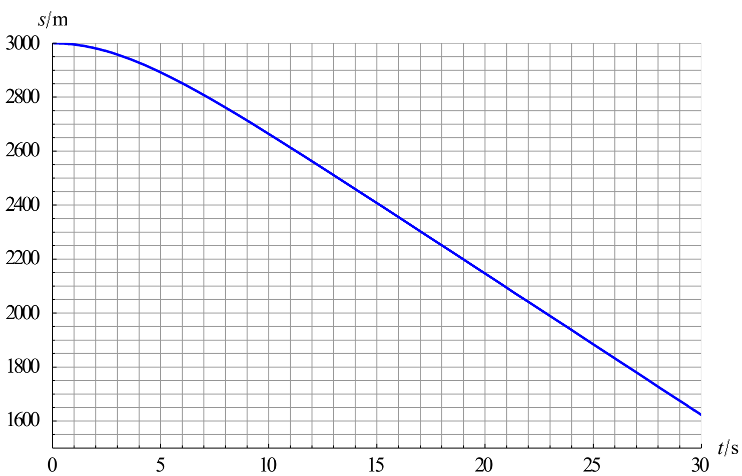
Worksheet on mechanics graphs

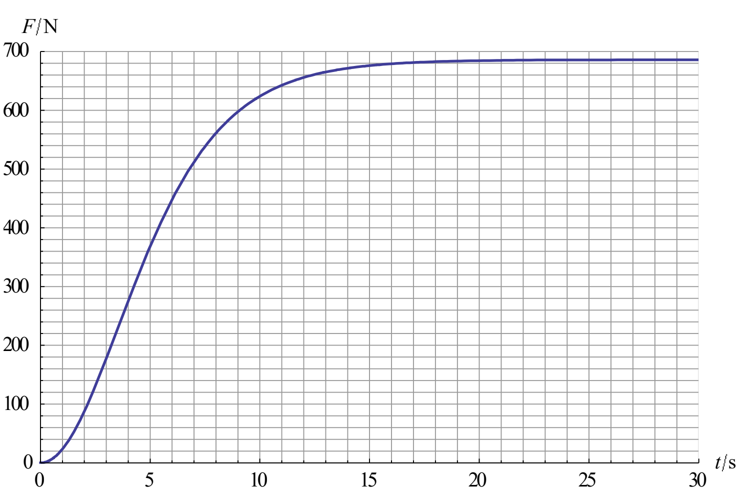
A parachutist P jumps out of a helicopter hovering at rest 3.0 km above the Earth's surface. Three graphs G1, G2 and G3 are available for the motion of P.



G1: The variation with time of the speed of P.



G2: The variation with time of the height of P from the ground before the parachute was opened.



G3: The variation with time of the drag force on P before the parachute was opened.

Use these graphs to answer the following questions:

(a)
- (i) What is the initial acceleration of P?
- (ii) When was the parachute opened?
- (iii) What is the average speed from the jump to the instant the parachute was opened?

(b)
- (i) What is the average acceleration experienced by P while the parachute was opening?
- (ii) What is the mass of P?
- (iii) What is the average force exerted on P by the parachute harness?
- (iv) The drag force is given by $F = kv^2$. Determine k.

(c) During the fall, P assumes two terminal speeds. Suggest why they are so different in magnitude.

(d) Draw a graph of acceleration vs time until just before the parachute is opened.

(e) Why can g be considered constant while P is falling?

(f) What energy has been converted to thermal energy until just before the parachute was opened?

and

(g) The helicopter has blades of length 15 m and a mass of 5200 kg. The density of air is 1.2 kg m^{-3}. The blades push air downward at speed v.
- (i) Explain how this allows the helicopter to hover stationary in air.
- (ii) Determine v.
- (iii) By what factor should v be increased so that the helicopter accelerates upwards with acceleration g?

Answers

(a)

(i) g, 9.8 m s^{-2}

(ii) t = 30 s.

(iii) Distance fallen is 3000 – 1625 = 1375 m in a time of 30 s and so average speed is 46 m s^{-1}.

(b)

(i) Velocity changed from 52 m s^{-1} to 6.0 m s^{-1} in a time of 5.0 s so the average acceleration is –9.2 m s^{-2}.

(ii) When the drag force is 680 N it is equal to the weight. Hence

$$m = \frac{680}{9.8} = 69 \quad \text{kg}.$$

(iii) $mg - T = ma \Rightarrow T = mg - ma = 680 - 69 \times (-9.2) = 1300\ \text{N}$

OR

$T - mg = ma \Rightarrow T = mg + ma = 680 + 69 \times 9.2 = 1300\ \text{N}$.

(iv) $k = \frac{680}{52^2} = 0.25 \quad \text{kg}\,\text{m}^{-1}$.

(c) Because the area providing the drag force is very different. It is the body of P for the large terminal speed and the much larger area of the parachute for the second lower terminal speed.

(d)

(e) $g = \frac{GM}{(R+h)^2}$ and the height h is negligible compared to the Earth radius R.

(f) Loss of potential energy at t = 30 s is $mgh = 69 \times 9.8 \times 1375 = 9.35 \times 10^5 \quad \text{J}$. Kinetic energy at t = 30 s is $\frac{1}{2}mv^2 = \frac{1}{2} \times 69 \times 52^2 = 9.33 \times 10^4 \quad \text{J}$. Hence energy transferred to thermal energy is $9.35 \times 10^5 - 9.33 \times 10^4 = 8.4 \times 10^5 \quad \text{J}$.

(g)

(i) The blades exert a force on the air pushing it down, so the air exerts a force of the same magnitude in the opposite direction on the blades by Newton's third law.

(ii) The momentum of the air pushed down in time Δt is $\Delta p = \Delta m\, v = (\rho \pi R^2 v \Delta t)v = \rho \pi R^2 v^2 \Delta t$. Hence the force is $F = \frac{\Delta p}{\Delta t} = \frac{\rho \pi R^2 v^2 \Delta t}{\Delta t} = \rho \pi R^2 v^2$. This equals the weight and so $\rho \pi R^2 v^2 = mg$ and thus $v = \sqrt{\frac{mg}{\rho \pi R^2}}$ giving v = 7.8 m s^{-1}.

(iii) The upward force must equal twice the weight. Hence $\rho \pi R^2 v_{\text{new}}^2 = 2mg$. This means that $\rho \pi R^2 v_{\text{new}}^2 = 2\rho \pi R^2 v^2$, i.e. $v_{\text{new}}^2 = 2v^2 \Rightarrow v_{\text{new}} = v\sqrt{2}$.